Knap sack with repetition

Given: \( n \) categories of items with weights \( w_1, w_2, \ldots, w_n \) and values \( v_1, v_2, \ldots, v_n \). A knapsack of capacity \( W \).

Find: A collection of items from the categories such that \( \sum v_i \) is maximized while \( \sum w_i \leq W \).

Subproblems: \( K(w) \), maximum value when a smaller bag would be full.

Goal: \( K(W) \), maximum value when bag full.

Initialization: \( K(0) = 0 \)

DAG Edges: \( K(w) = \max_{i: w_i \leq w} \{ K(w-w_i) + v_i \} \)

Linearized order: \( 1 \ldots W \)
Knapsack Algorithm

\[ k(0) = 0 \]

for \( w = 1 \ldots \text{W} \):

\[ k(w) = \max_{i : w_i \leq w} \left\{ k(w - w_i) + v_i \right\} \]

return \( k(\text{W}) \)

\( O(nW) \)

\( 2^{\log_2(W)} \)

pseudo-polynomial