Set Cover

Given: \( B = \) set of elements
\( S_i = \) set of elements "covered" by element \( i \)
\( S_i \subseteq B \)

Find: A selection of sets \( S_i \) that completely "cover" \( B \).

\( S_i \cup S_j \cup \ldots \cup S_k = B \)

Cost = number of sets \( S_i \) used.

Find minimum cost solution.
Example:

Alien detection stations:

Each station can detect alien warships in its location, and in the location of its connected neighbors.

Reduce power consumption by only turning on the minimal set of detectors.

\[ B = \{ a, b, c, d, e, f, g, h, i \} \]

\[ S_a = \{ a, d \} \]
\[ S_b = \{ b, c, e \} \]
\[ S_c = \{ b, c, e \} \]
\[ S_d = \{ a, d, g \} \]
\[ S_e = \{ b, c, e, f, g \} \]

\[ S_f = \{ e, f \} \]
\[ S_g = \{ d, e, g \} \]
\[ S_h = \{ e, h, i \} \]
\[ S_i = \{ h, i \} \]

a solution: \[ S_e, S_d, S_h \]

Is it minimal?
set cover

How to find optimal solution?
Set Cover

How to find greedy (non-optimal) solution?