1. Write the truth table for the proposition \( \neg(r \rightarrow \neg q) \vee (p \land \neg r) \).

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2. Write a proposition equivalent to \( p \rightarrow q \) using only \( p, q, \neg \) and the connective \( \land \).

\[
(\neg p \lor q) \land \neg (\neg p \lor q) \land \neg (p \land \neg q)
\]

In question 3 below suppose the variable \( x \) represents students and \( y \) represents courses, and:

\( T(x, y) \): \( x \) is taking \( y \).

Write the statement in good English without using variables in your answers.

3. \( \exists x \forall y T(x, y) \).

See Key

4. Consider the following theorem: If \( x \) is an odd integer, then \( x + 2 \) is odd. Give a direct proof of this theorem

See Key
5. Find three subsets of \( \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \) such that the intersection of any two has size 2 and the intersection of all three has size 1. HINT: Use a Venn Diagram.

6. Suppose \( U = \{1, 2, \ldots, 9\} \), \( A = \) all multiples of 2, \( B = \) all multiples of 3, and \( C = \{3, 4, 5, 6, 7\} \). Find \( C - (B - A) \).

7. Give an example of a function \( f: \mathbb{R} \rightarrow \mathbb{R} \) that is Onto but not 1-1. Remember that Onto means every element in the codomain gets hit at least once, and 1-1 means that no element in the codomain gets hit more than once. So you need a function where they all get hit at least once, and at least 1 of them gets hit more than once.

8. Suppose Set A has 5 elements and set B has 3 elements. How many different functions map the elements of A onto the elements of B?

9. Find a formula that generates the following sequence: 15, 20, 25, 30, 35, \ldots

10. List all positive integers between 2 and 30 that are relatively prime to 20.

11. Find the hexadecimal representation of \( ABC_{16} + 2F_{16} \).

12. What sequence of pseudorandom numbers is generated using the linear congruential generator \( x_{n+1} = (5x_n + 3) \mod 13 \) with seed \( x_0 = 1 \)? List the numbers until they start repeating.
13. Convert \((204)_{10}\) to base 2.

\[
204 = 2 \cdot 102 + 0  \\
102 = 2 \cdot 51 + 0  \\
51 = 2 \cdot 25 + 1  \\
25 = 2 \cdot 12 + 1  \\
12 = 2 \cdot 6 + 0  \\
6 = 2 \cdot 3 + 0  \\
3 = 2 \cdot 1 + 1  \\
i = 2 \cdot 0 + 1
\]

**Answer:** 11001100₂

14. Use the Euclidean Algorithm to find \(\text{gcd}(900, 140)\).

\[
900 = 140 \cdot 6 + 60  \\
140 = 60 \cdot 2 + 20  \\
60 = 20 \cdot 3 + 0
\]

In questions 15 and 16 below suppose you have 30 books (15 novels, 10 history books, and 5 math books). Assume that all 30 books are different.

15. In how many ways can you gather a bag of three history books and seven novels to give to a friend?

\[\binom{10}{3} \cdot \binom{15}{7}\]

16. In how many ways can you put the 30 books in a row on a shelf if the novels are on the left, the math books are in the middle, and the history books are on the right?

\[15! \cdot 5! \cdot 10!\]

In questions 17 and 18 below assume that you have a bowl containing hard candies: 50 cherry, 50 strawberry, 40 orange, 70 lemon, and 40 pineapple. Assuming that the pieces of each flavor are identical.

17. How many handfulls of 15 are possible?

\[\binom{51}{15} = \binom{19}{15}\]

**Note:** The supply of each flavor does not matter.

18. How many handfulls of 15 are possible with at least two pieces of each flavor?

\[\text{10 of your 15 choices are made for you.}\]

\[\binom{5}{5} = \binom{9}{5}\]

19. Find the next largest permutation in lexicographic order after 3254671.

\[\text{Target replaced with one just bigger (7) is 3254716.}\]
20. What is the probability that a fair coin lands Heads 6 times in a row?

\[ \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \]

21. A group of ten women and ten men are in a room. A committee of four is chosen at random. Find the probability that the committee consists only of women.

**Answer Key**

\[ \frac{C(10,4)}{C(20,4)} \]

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2. \( \neg(p \land \neg q) \).

3. Some student is taking every course.

4. Let \( x = 2k + 1 \). Therefore \( x + 2 = 2k + 1 + 2 = 2(k + 1) + 1 \), which is odd.

5. For example, \{1,2,3\}, \{2,3,4\}, \{1,3,4\}.

6. \{4,5,6,7\}.

7. \( f(x) = (x)^*(x-2)^*(x+2) \), for example

8. \( 3^*3^*3^*3 \).

9. \( a_n = 5(n + 2) \).

10. 1,3,7,9,11,13,17,19,21,23,27,29.

11. \( (DBI)_{16} \).

12. 1,8,4,10,repeat

13. 1100 1100.

14. 20.

15. \( \binom{10}{3} \binom{15}{7} \).

16. \( 15! \cdot 5! \cdot 10! \).

17. \( \binom{19}{4} \).

18. \( \binom{9}{4} \).
For question 1 below, suppose the variable $x$ represents people, and:

$F(x)$: $x$ is friendly, \hspace{1cm} $T(x)$: $x$ is tall.

Write the statement using these predicates and any needed quantifiers.

1. All tall people are friendly.

Note: The answer is NOT $\forall x \, (T(x) \land F(x))$ everyone is tall and friendly.

For question 2 below, suppose the variable $x$ represents students and $y$ represents courses, and:

$T(x,y)$: $x$ is taking $y$.

Write the statement in good English without using variables in your answers.

2. $\forall x \exists y T(x,y)$. All students are taking at least one course.

3. Multiply $4103_5$ and $2230_5$, keeping the result in base 5.

$$
\begin{array}{c}
\phantom{0}x&4&1&0&3_5 \\
&2&2&3&0_5 \\
\hline \\
&\phantom{0}1&3&2&1_5 \\
&2&0&3&0_5 \\
&\phantom{0}9&2&1&9&1 \\
\end{array}
$$

4. Convert $10000_{10}$ to base 2.

5. Convert $(123)_0$ to base 10.

6. Use the Euclidean Algorithm to find $\gcd(580,50)$.

7. Find the inverse of 9 (mod 32), using the algorithm from class. Show your work.
8. What sequence of pseudorandom numbers is generated using the linear congruential
generator \( x_{n+1} = (3x_n + 2) \mod 13 \) with seed \( x_0 = 1 \)? List the numbers until they start
repeating.

\[
\begin{align*}
3 \cdot 1 + 2 &= 5 \\
(3 \cdot 5 + 2) \mod 13 &= 4 \\
(3 \cdot 4 + 2) \mod 13 &= 1 \\
\end{align*}
\]

For questions 9-10 below, suppose that a “word” is any string of exactly 7 letters from the 26
letter alphabet, with repeated letters allowed.

9. How many words are there?

\[
\begin{array}{cccccc}
2 & 6 & 2 & 6 & 2 & 6 \\
2 & 6 & 2 & 6 & 2 & 6 \\
\end{array}
\]

10. How many words contain exactly 2 vowels?

- List the next 4 combinations in lexicographical order after “1257”, when taking 4 items

11. at a time from the set \{1,2,3,4,5,6,7\}.

\[
\begin{array}{cccc}
1 & 2 & 6 & 7 \\
1 & 3 & 4 & 5 \\
1 & 3 & 4 & 6 \\
1 & 3 & 4 & 7 \\
\end{array}
\]

For questions 12-14 below, suppose Baskin Robins has 31 flavors of ice cream. You are
supposed to choose flavors, where order does NOT matter:

12. How many ways can you pick 5 flavors, where repetition is allowed?

\[
\binom{31}{5} = \binom{31}{5}
\]

13. How many ways can you pick 10 flavors, where repetition is allowed – given the
following restraints: Peppermint must be one of them, cherry must not be, at least 2
must be strawberry, and at most 2 must be vanilla?

\[
\binom{31}{10} \\
\binom{31}{9} \text{ Peppermint chosen} \\
\binom{30}{9} \text{ No cherry} \\
\binom{30}{7} \text{ 2 Strawberry chosen} \\
\binom{30}{7} - \binom{30}{4} \text{ Subtract bad 3 or more vanilla}
\]

14. How many ways can you pick 5 flavors, where repetition is NOT allowed?

\[
\binom{31}{5}
\]
15. What is the probability of being dealt "3 of a kind" when given 5 cards from a standard deck of 52 cards? For simplicity, you may include anything more than "3 of a kind" such as '4 of a kind" and "3 of one and 2 of another." Show your thinking.

\[
\frac{13 \cdot 4 \cdot \binom{49}{2}}{\binom{52}{5}} \quad 13 \text{ ranks to choose from} \\
4 \text{ ways to get 3 of 4 suits} \\
\binom{49}{2} \text{ ways to fill in last 2 cards}
\]

16. Find the prime factorization of 90.

\[2 \cdot 3 \cdot 3 \cdot 5\]

For questions 17-18 below, suppose nine people (Ann, Ben, Cal, Dot, Ed, Fran, Gail, Hal, and Ida) are in a room. Five of them stand in a row for a picture. \(\binom{9}{5}\)

17. In how many ways can this be done if a taller person must be to the left of a shorter person? (Assume all have different heights.)

\[\binom{9}{5} \quad \text{once you choose which 5, the order is fixed.}\]

18. In how many ways can this be done if Ann and Ben refuse to be in a picture together? (That is, a given picture may contain Ann, or Ben, or neither, but never both.)

\[\text{Bad is Ann and Ben both in picture} = 5 \cdot 4 \cdot \binom{7}{3} \quad \text{Ann or Ben rest}\]

\[\text{Good} = \text{Total} - \text{Bad} = \binom{9}{5} - 5 \cdot 4 \cdot \binom{7}{3}\]

19. When dealing with Boolean variables, there are several Boolean operators we know of, such as And, Or, Implies, Exclusive Or, Nand, Nor, etc. How many unique binary operators could we form? (That is, if we create another operator that has the same truth table values as the And operator, then it would Not be unique.)

\[
\text{The operator controls 4 Boolean variables} \\
\text{So there are } 2^4 = 16 \text{ max operators,}
\]

20. How many bit strings of length 10 contain at most two 1s?

\[\binom{10}{2} + \binom{10}{1} + \binom{10}{0}\]

21. Add (4133)_5 and (4234)_5, keeping the result in base 5.

\[
\begin{array}{c c c c}
4 & 1 & 3 & 3 \\
+ & 4 & 2 & 3 \\
\hline
1 & 3 & 4 & 2 & 2
\end{array}
\]

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